Exercise 1

The evolution matrix of a semiconductor laser can be expressed as follows,

$$J = \begin{pmatrix} -\frac{1}{\tau_s} - G_N S & -\frac{1}{\tau_p} \\ G_N S & 0 \end{pmatrix}$$

with G_N the differential gain, τ_S the carrier lifetime, τ_p the cavity photon lifetime, and S the photon density.

Q1. Why the evolution matrix has a dimension of two?
Q2. Express the characteristic polynomial.
Q3. Show that the eigenvalues are

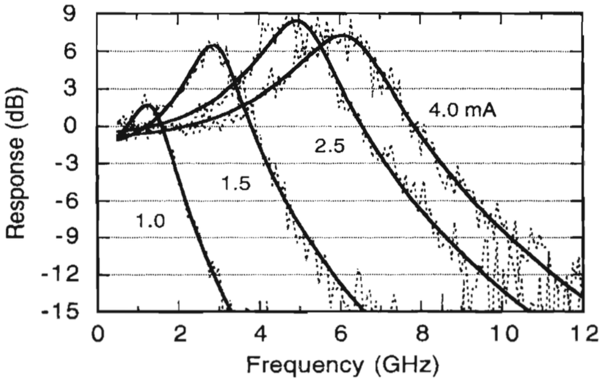
$$\lambda_{\pm} = -\Gamma \pm \sqrt{\Gamma^2 - \omega_R^2}$$

Express Γ and ω_R as a function of G_N, τ_S, τ_p and S.

Q4. Give the physical meanings associated to Γ and ω_R?

Q5. Assuming $\Gamma << \omega_R$, show that the eigenvalues are complex numbers. Conclusions.

Q6. The figure below displays the modulation response of a directly-modulated semiconductor laser. The modulation response is measured for various pump currents ranging from 1 to 4 mA.



Briefly discuss on the importance of this experimental data set for high-speed applications. Use Q4 & Q5 to explain the shape of the modulation response.

Exercise 2

One of the standard forms of single-mode laser equations is derived from the Maxwell–Bloch equations based on the semi-classical theory, where the electric field is described by the Maxwell's equations, the macroscopic atomic polarization is introduced by using Schrödinger equation, and the phenomenological atomic and photon decays are introduced. Assuming a homogeneously broadened single-mode laser (on resonance), rate equations can be simplified as follows,

$$\frac{dE}{dt} = -\varkappa[E(t) + AP(t)]$$

$$\frac{dP}{dt} = -P(t) - E(t)D(t)$$

$$\frac{dD}{dt} = \gamma[1 - D(t) + E(t)P(t)]$$

where E(t) is the electric field, P(t) the atomic polarization, and D(t) the population inversion. In the above equations, time is normalized by the decay rate of the atomic polarization $\gamma_{\perp}$ whereas $\gamma_{\parallel}$ is the decay rate of the population inversion ($\gamma = \gamma_{\parallel}/\gamma_{\perp}$), and κ the decay rate of the electric field in the laser cavity.

Q1. Find the new set of dynamical rate equations for the following cases
(a) Class C lasers ($\kappa \sim \gamma_{\perp} \sim \gamma_{\parallel}$);
(b) Class B lasers ($\gamma_{\perp} >> \kappa >> \gamma_{\parallel}$);
(c) Class A lasers ($\kappa << \gamma_{\perp} \sim \gamma_{\parallel}$).

Explain your methodology in a few sentences. For each case, remind the variables used to describe the dynamics, the conditions (rates). What is the class of a semiconductor laser?

Hint: A dynamical variable relaxing to its steady-state value much faster than the others can be adiabatically eliminated.

Q2. Which of the aforementioned laser classes satisfy the conditions for chaos generation? Explain why is that and link your conclusions to the Poincaré-Bendixson theorem?